

Nonperturbative features of the axial current

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ABSTRACT: In this paper we study the nonperturbative structure of the axial current and evaluate the two-point distribution amplitudes $\int d\xi e^{-iq \cdot \xi} \langle 0 | \bar{\psi}(x) \Gamma \psi(y) J_\mu^5(\xi) | 0 \rangle$ in the framework of the instanton vacuum model in the leading order in $\mathcal{O}(N_c)$. We perform a direct numerical test of the relations between the axial current and the pion distribution amplitudes, imposed by PCAC, and found excellent agreement.

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1 Introduction

According to the Standard Model, the interaction of W - and Z -bosons with quarks has a $V - A$ structure. While the hadronic structure of the vector current has been well established in photon-hadron and photon-nuclear processes, the structure of the axial current is less known. Due to the spontaneous breaking of the chiral symmetry the structure of the axial current is different from the vector case. In order to describe the longitudinal component of the axial current a widely used tool is a phenomenological PCAC relation, which was proposed in the pre-QCD era [1–4], and relates the longitudinal part of the interaction amplitude of the axial current with that of the pion [5]. However, PCAC is not applicable to a description of the transversely polarized axial current.

The standard description of high-energy processes is based on factorization [6] of the corresponding amplitude to a process-dependent hard part, evaluated in pQCD, and a target-dependent soft distribution amplitude (DA), which is either extracted from fits to data, or evaluated relying on low-energy models for the target.

In this paper we evaluate the two-point quark distribution amplitudes (DAs) of the axial current in the framework of the instanton vacuum model (see [7–9] and references therein). The important advantage of this model is that it has a built-in dynamically broken

chiral symmetry and as a consequence, reproduces the low-energy chiral structure of QCD. Recently this model was used for evaluation of the vector current hadronic structure [10, 11], pion distribution amplitudes [12–17], correlators of vector and axial currents [10, 18, 19], as well as different low-energy constants. The nonperturbative structure of the axial current is particularly important for processes which include soft kinematics or large distance between quarks.

The paper is organized as follows. In Section 2 we briefly overview the basic elements of the instanton vacuum model (IVM), which is used for further evaluations. In Section 3 we give definitions for the DAs, and in Sections 3.1–3.8 provide analytic expressions and numerical results for each of the DAs. In Section 4 we directly check the accuracy of the PCAC relation within the IVM model. We summarize observations and make conclusions in Section 5.

2 Instanton vacuum model

The central object of the model is the effective action for the light quarks in the instanton vacuum, which in the leading order in N_c has the form [8, 9]

$$S = \int d^4x \left(\frac{N}{V} \ln \lambda + 2\Phi^2(x) \bar{\psi} (\hat{p} + \hat{v} + \hat{a}\gamma_5 - m - c\bar{L}f \otimes \Phi \cdot \Gamma_m \otimes fL) \psi \right) \quad (2.1)$$

where Γ_m is one of the matrices, $\Gamma_m = 1, i\vec{\tau}, \gamma_5$, or $i\vec{\tau}\gamma_5$; ψ and Φ are the fields of constituent quarks and mesons respectively; N/V is the density of the instanton gas; $\hat{v} \equiv v_\mu \gamma^\mu$ is the external vector current corresponding to the photon; L is the gauge factor,

$$L(x, z) = P \exp \left(i \int_z^x d\zeta^\mu (v_\mu(\zeta) + a_\mu(\zeta) \gamma_5) \right), \quad (2.2)$$

$$\bar{L}(x, z) = \gamma_0 L(x, z)^\dagger \gamma_0 \quad (2.3)$$

which provides the gauge covariance of the action [20, 21]; and $f(p)$ is the Fourier transform of the zero-mode profile in the single-instanton background. In this paper we employ the dipole-form [8] for the parameterization of the formfactor,

$$f(p) = \frac{L^2}{L^2 - p^2} \quad (2.4)$$

with $L \sim 850$ MeV. It is worth mentioning that the distribution amplitudes, due to their definition as nonlocal quark operators with light-cone separation, are much more sensitive to the choice of the formfactor compared with various vacuum condensates. Varying the formfactor $f(p)$ completely different results were obtained for the leading-twist pion DA in [12, 13, 22].

In the leading order in N_c , we have the same Feynman rules as in perturbative theory, but with a momentum-dependent quark mass $\mu(p)$ in the quark propagator

$$S(p) = \frac{1}{\hat{p} - \mu(p) + i0}. \quad (2.5)$$

The running mass of the constituent quark has a form

$$\mu(p) = m + M f^2(p), \quad (2.6)$$

where $m \approx 5$ MeV is the current quark mass, $M \approx 350$ MeV is the dynamical mass generated by the interaction with the instanton vacuum background. Due to presence of instantons the vector current - quark coupling is also modified,

$$\hat{v} \equiv v_\mu \gamma^\mu \Rightarrow \hat{V} = \hat{v} + \hat{V}^{nonl}, \quad (2.7)$$

$$\hat{a} \equiv a_\mu \gamma^\mu \Rightarrow \hat{A} = \hat{a} + \hat{A}^{nonl}, \quad (2.8)$$

In addition to the vertices present in perturbative QCD, the model has nonlocal terms with higher-order couplings of currents and mesons. The exact expressions for the nonlocal terms $\hat{V}^{nonl}, \hat{A}^{nonl}$ depend on the choice of the path in (2.2), and one can find in the literature different results [10, 11, 13, 23]. This ambiguity in the DAs is important for the transversely polarized, but is negligibly small for the longitudinally polarized axial current. In what follows we use the parameterizations,

$$\hat{V}_{nonl} = v_\mu \left(iM \frac{p_1^\mu + p_2^\mu}{p_1^2 - p_2^2} \left(f(p_1)^2 - f(p_2)^2 \right) \right), \quad (2.9)$$

$$\hat{A}_{nonl} = a_\mu \left(iM \frac{p_1^\mu + p_2^\mu}{p_1^2 - p_2^2} \left(f(p_1) - f(p_2) \right)^2 \right), \quad (2.10)$$

where p_1, p_2 are the momenta of the initial and final quarks.

3 Distribution amplitudes for the axial current

The DAs of the axial current are defined via 3-point correlators,

$$\Psi_\mu \sim \int d^4\xi e^{-iq \cdot \xi} \langle 0 | \bar{\psi}(y) \Gamma \psi(x) J_\mu^5(\xi) | 0 \rangle, \quad (3.1)$$

where x and y are light-cone coordinates of the quark and antiquark, q is the momentum flowing through the axial current and Γ is one of the Dirac matrices, as was defined in (2.1). The structure of the axial current is different from the vector one because of the spontaneous chiral symmetry breaking and existence of near-massless pions. In particular, the axial current can fluctuate into a pion prior to the production of a $\bar{q}q$ pair. Therefore, the correlator (3.1) consists of two terms, schematically presented in the Figure 1.

One term comes from the combined contribution of the intermediate heavy states (a_1 meson, 3π , etc.), and the other one represents fluctuations of the axial current into a pion. The chiral symmetry embedded into the model relates the two terms as,

$$\Psi_\mu = \Psi_\mu^{(bulk)} + \Psi_\mu^{(pion)} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2 - m_\pi^2} \right) \Psi_\nu^{(bulk)}. \quad (3.2)$$

This form of the DA explicitly satisfies PCAC. In what follows we concentrate on the part of the amplitude presented in the dispersion relation for the amplitude by the bulk of heavy

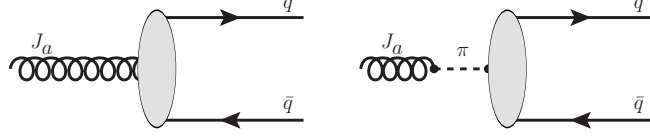


Figure 1. The DA contains two terms corresponding to either intermediate heavy axial states (left), or to a pion (right), which we label by *bulk* or *pion* respectively.

states excluding the pion pole (left pane of Figure 1) [24, 25], tacitly assuming that the full DAs are given by Eq. (3.2). We define these DAs as,

$$\int d^4x e^{-iqx} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_\mu \gamma_5 \psi \left(\frac{z}{2} \right) J_\beta^5(x) \right| 0 \right\rangle = if_A \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \times \quad (3.3)$$

$$\times \left(\frac{p_\mu z_\beta}{p \cdot z} \Phi_{||}(\alpha) + g_{\mu\beta}^\perp g_\perp^{(a)}(\alpha) + \frac{z_\mu z_\beta}{(p \cdot z)^2} g_3(\alpha) \right),$$

$$\int d^4x e^{-iqx} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_\mu \psi \left(\frac{z}{2} \right) J_\beta^5(x) \right| 0 \right\rangle = -if_A \epsilon_{\mu\beta\rho\sigma} p_\rho n_\sigma \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \frac{g_\perp^{(v)}(\alpha)}{4} \quad (3.4)$$

$$\int d^4x e^{-iqx} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \sigma_{\mu\nu} \gamma_5 \psi \left(\frac{z}{2} \right) J_\beta^5(x) \right| 0 \right\rangle = f_A \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \left(\left(g_{\beta\mu}^\perp p_\nu - g_{\beta\nu}^\perp p_\mu \right) \Phi_\perp(\alpha) \right. \quad (3.5)$$

$$+ \frac{z_\beta}{(p \cdot z)^2} (p_\mu z_\nu - p_\nu z_\mu) h_{||}^{(t)}(\alpha)$$

$$+ \left. \frac{1}{2} \left(g_{\beta\mu}^\perp z_\nu - g_{\beta\nu}^\perp z_\mu \right) \frac{1}{p \cdot z} h_3(\alpha) \right),$$

$$\int d^4x e^{-iqx} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_5 \psi \left(\frac{z}{2} \right) J_\beta^5(x) \right| 0 \right\rangle = f_A n_\beta \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \frac{h_{||}^{(p)}(\alpha)}{2}, \quad (3.6)$$

where q is the 4-momentum carried by the axial current; u and $\bar{u} \equiv 1 - u$ are the fractional light-cone momenta carried by the quark and antiquark; $z = x - y$; p_μ is the “positive direction” vector on the light-cone, n_μ is the “negative direction” vector on the light-cone, with normalization conditions $p \cdot n = 1$, $p^2 = n^2 = 0$. Transverse dimensions are chosen in such a way that the vector q has only components in the p, n - plane. Without any loss of generality, in what follows we choose a system where $q_+ = q \cdot n = 1$. The normalization constant f_A is a dimensional parameter introduced in order to make the distribution amplitudes dimensionless. Its value is fixed by the condition

$$\int_0^1 d\alpha \Phi_{||}(\alpha, Q^2 = 0) = 1. \quad (3.7)$$

If we define an “effective” axial meson state vector $|A^{(\lambda)}(q)\rangle$ as

$$|A^{(\lambda)}(q)\rangle = \int d^4x e^{-iq \cdot x} e_{\beta}^{(\lambda)}(q) J_{\beta}(x) |0\rangle, \quad (3.8)$$

where the polarization vectors $e^{(\lambda)}$ are defined as [26]

$$e_{\mu}^{(\lambda=||)} = e^{(\lambda)} \cdot n \left(p_{\mu} - \frac{q^2}{2} n_{\mu} \right), \quad (3.9)$$

$$p \cdot e_{\mu}^{(\lambda=\perp)} = n \cdot e_{\mu}^{(\lambda=\perp)} = 0, \quad (3.10)$$

$$|e^{(\perp)}(q)|^2 = -1, \quad (3.11)$$

then we may rewrite Eqns. (3.12-3.15) in a standard form as distribution amplitudes of the effective axial meson state [27],

$$\begin{aligned} \langle 0 | \bar{\psi}(y) \gamma_{\mu} \gamma_5 \psi(x) | A(q) \rangle &= i f_A \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \times \\ &\times \left(p_{\mu} \frac{e^{(\lambda)} \cdot z}{p \cdot z} \Phi_{||}(\alpha) + e_{\mu}^{(\lambda=\perp)} g_{\perp}^{(a)}(\alpha) + z_{\mu} \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^2} g_3(\alpha) \right) \end{aligned} \quad (3.12)$$

$$\langle 0 | \bar{\psi}(y) \gamma_{\mu} \psi(x) | A(q) \rangle = -i f_A \epsilon_{\mu\nu\rho\sigma} e_{\nu}^{(\lambda)} p_{\rho} z_{\sigma} \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \frac{g_{\perp}^{(v)}(\alpha)}{4} \quad (3.13)$$

$$\begin{aligned} \langle 0 | \bar{\psi}(y) \sigma_{\mu\nu} \gamma_5 \psi(x) | A(q) \rangle &= f_A \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \left(\left(e_{\mu}^{(\lambda=\perp)} p_{\nu} - e_{\nu}^{(\lambda=\perp)} p_{\mu} \right) \Phi_{\perp}(\alpha) + \right. \\ &+ \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^2} (p_{\mu} z_{\nu} - p_{\nu} z_{\mu}) h_{||}^{(t)}(\alpha) \\ &+ \left. \frac{1}{2} \left(e_{\mu}^{(\lambda)} z_{\nu} - e_{\nu}^{(\lambda)} z_{\mu} \right) \frac{1}{p \cdot z} h_3(\alpha) \right), \end{aligned} \quad (3.14)$$

$$\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | A(q) \rangle = f_A e^{(\lambda)} \cdot n \int_0^1 d\alpha e^{i(0.5-\alpha)p \cdot z} \frac{h_{||}^{(p)}(\alpha)}{2}. \quad (3.15)$$

The distribution amplitudes $\Phi_{||}(u), \Phi_{\perp}(u)$ are of twist-2; $g_{\perp}^{(a)}, g_{\perp}^{(v)}, h_{||}^{(t)}, h_{||}^{(p)}$ are of twist-3; g_3, h_3 are of twist-4. Chiral parity: all wave functions in (3.12), (3.13) are chiral even, all wave functions in (3.14), (3.15) are chiral odd.

Next step is a numerical evaluation of different DAs.

3.1 Evaluation of $\Phi_{||}(\alpha, q^2)$

In order to extract this function, we have to take a convolution of (3.12) with $p_\beta n_\mu$, which yields

$$\begin{aligned}\Phi_{||}(\alpha, q^2) &= \frac{1}{if_A} \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_+ \gamma_5 \psi \left(\frac{z}{2} \right) J_-^5(x) \right| 0 \right\rangle \quad (3.16) \\ &= \frac{8N_c}{f_A} \int \frac{dl^- d^2l_\perp}{(2\pi)^4} \left[\frac{\mu(l)\mu(l+q) + l_\perp^2 + \alpha\bar{\alpha}q^2}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\ &\quad \left. - \frac{M(f(l+q) - f(l))^2 (2l^- + q^2\alpha) (\mu(l)\bar{\alpha} + \mu(l+q)\alpha)}{((l+q)^2 - l^2) (l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+} \quad (3.17)\end{aligned}$$

Obviously, this function is symmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. The normalization condition (3.7) gives for f_A

$$f_A = 8N_c \int \frac{d^4l_\perp}{(2\pi)^4} \left[\frac{\mu^2(l) + l_\perp^2}{(l^2 + \mu^2(l))^2} - \frac{M(f'(l))^2 l^-}{l^2 (l^2 + \mu^2(l))^2} \right] \quad (3.18)$$

As is discussed in the Section 4, if the PCAC relation is valid, then $f_A \approx \sqrt{2}f_\pi^2$.

In Figure 2 the DA $\Phi_{||}(\alpha)$ is plotted as function of α for several values of Q^2 .

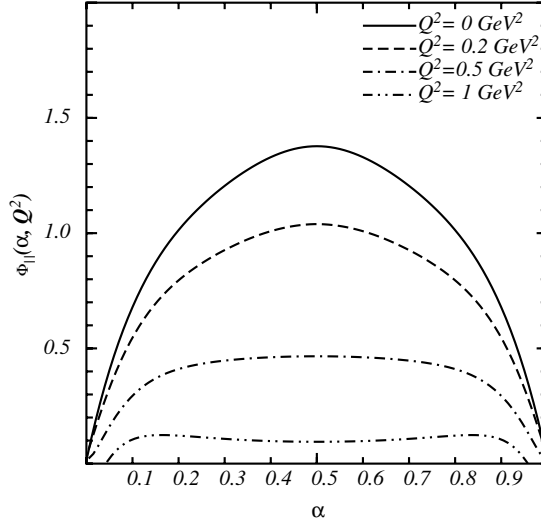


Figure 2. Distribution amplitude $\Phi_{||}(\alpha)$ vs. α for several values of Q^2 .

3.2 Evaluation of $\Phi_{\perp}(\alpha, q^2)$

In this section we evaluate the function

$$\begin{aligned}
\Phi_{\perp}(\alpha, q^2) &= \frac{1}{4f_A} (g_{\beta\mu}n_{\nu} - g_{\beta\nu}n_{\mu}) \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n \right) \sigma_{\mu\nu} \gamma_5 \psi \left(\frac{z}{2}n \right) J_{\beta}^5(x) \right| 0 \right\rangle \\
&= \frac{8N_c}{f_A} \int \frac{dl_{-} d^2l_{\perp}}{(2\pi)^3} \left[\frac{-\alpha\mu(l+q) + \bar{\alpha}\mu(l)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
&\quad \left. + \frac{l_{\mu\perp}^2}{(l+q)^2 - l^2} \frac{M(f(l+q) - f(l))^2}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^{+} = -\alpha q^{+}}
\end{aligned} \tag{3.19}$$

This function is antisymmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. In Figure 3 the distribution amplitude is shown for several values of Q^2 .

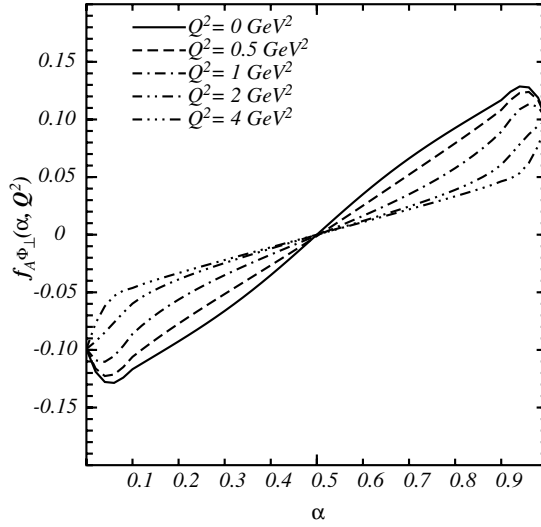


Figure 3. Distribution amplitude $\Phi_{\perp}(\alpha)$ for several values of Q^2 .

3.3 Evaluation of $g_{\perp}^{(a)}(\alpha, q^2)$

In order to extract this function, we have to take a convolution of (3.12) with $\frac{1}{2}g_{\beta\mu}^{\perp}$, which yields

$$\begin{aligned}
g_{\perp}^{(a)}(\alpha, q^2) &= \frac{8N_c}{if_A} \int \frac{dl_{-} d^2l_{\perp}}{(2\pi)^4} \left[\frac{\mu(l)\mu(l+q) + (2\alpha - 1)l_{-}q_{+} + \alpha\frac{q^2}{2}}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
&\quad \left. + \frac{l_{\mu\perp}^2}{(l+q)^2 - l^2} \frac{M(f(l+q) - f(l))^2 (\mu(l+q) - \mu(l))}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^{+} = -\alpha q^{+}}
\end{aligned} \tag{3.20}$$

This function is symmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. In Figure 4 the distribution amplitude is shown for several values of Q^2 .

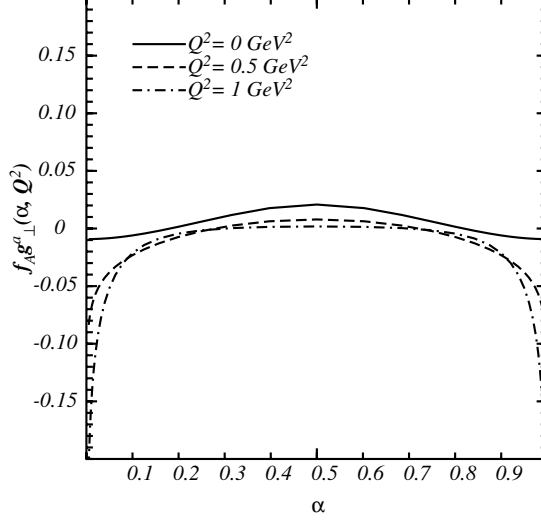


Figure 4. Distribution amplitude $g_{\perp}^a(\alpha)$ for several values of Q^2 .

3.4 Evaluation of $g_{\perp}^{(v)}(\alpha, q^2)$

In order to extract $g_{\perp}^{(v)}(u)$, we have to take a convolution of (3.13) with $\epsilon_{\mu\beta\rho'\sigma'}p_{\rho'}n_{\sigma'}$, which yields

$$\begin{aligned}
 g_{\perp}^{(v)}(\alpha, q^2) &= -\frac{2\epsilon_{\mu\beta\rho\sigma}p_{\rho}n_{\sigma}}{if_A} \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_{\mu} \psi \left(\frac{z}{2} \right) J_{\beta}^5(x) \right| 0 \right\rangle \\
 &= \frac{16N_c}{f_A} \int \frac{dl^- d^2 l_{\perp}}{(2\pi)^4} \left[\frac{\alpha \frac{q^2}{2} + l_{-} q_{+}}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^{+} = -\alpha q^{+}}. \quad (3.21)
 \end{aligned}$$

This function is antisymmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. In Figure 5 the distribution amplitude is shown for several values of Q^2 .

3.5 Evaluation of $h_{\parallel}^{(t)}(\alpha, q^2)$

In this section we evaluate the function

$$\begin{aligned}
 h_{\parallel}^{(t)}(\alpha, q^2) &= \frac{(p_{\nu}n_{\rho} - p_{\rho}n_{\nu})p_{\beta}}{2f_A} \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n \right) \sigma_{\nu\rho} \gamma_5 \psi \left(\frac{z}{2}n \right) J_{\beta}^5(x) \right| 0 \right\rangle \\
 &= -\frac{8N_c}{f_A} \int \frac{dl^- d^2 l_{\perp}}{(2\pi)^4} e^{-i\vec{l}_{\perp} \cdot \vec{r}_{\perp}} \left[\frac{\mu(l+q)l_{-} + \mu(l) \left(l_{-} + \frac{q^2}{2} \right)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
 &\quad \left. + \frac{2M \left(l_{-} + \frac{q^2}{4} \right) \left(l_{-} + \alpha \frac{q^2}{2} \right) (f(l+q) - f(l))^2}{((l+q)^2 - l^2) (l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^{+} = -\alpha q^{+}} \quad (3.22)
 \end{aligned}$$

This function is antisymmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. In Figure 6 the distribution amplitude is shown for several values of Q^2 .

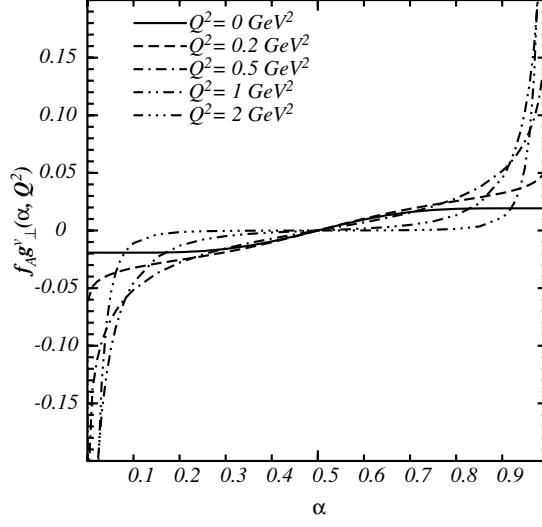


Figure 5. Distribution amplitude $g_{\perp}^v(\alpha)$ for several values of Q^2 .

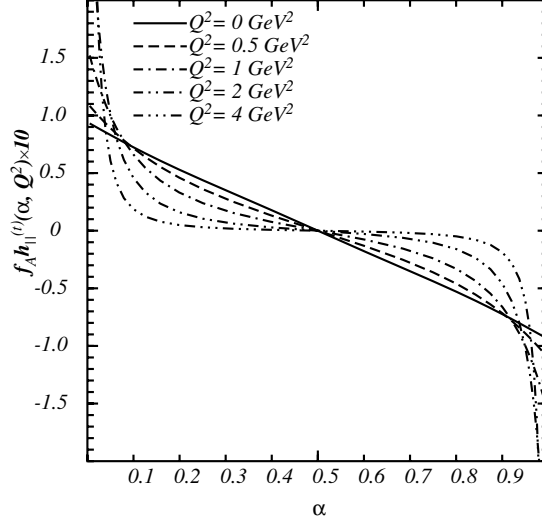


Figure 6. Distribution amplitude $h_{\parallel}^{(t)}(\alpha)$ for several values of Q^2 .

3.6 Evaluation of $h_{||}^{(p)}(\alpha, q^2)$

In order to extract $h_{||}^{(p)}(u)$, we have to take a convolution of (3.15) with $2p_\beta/f_A$, which yields

$$\begin{aligned}
h_{||}^{(p)}(\alpha, q^2) &= -\frac{2p_\beta}{f_A} \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_5 \psi \left(\frac{z}{2} \right) J_\beta^5(x) \right| 0 \right\rangle \\
&= -\frac{16N_c}{f_A} \int \frac{dl^- d^2l_\perp}{(2\pi)^4} e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} \left[\frac{(\mu(l) - \mu(l+q)) \left(l^- + \alpha \frac{q^2}{2} \right)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
&\quad \left. + \frac{\left(l_\perp^2 + (1-2\alpha)l^- - \alpha \frac{q^2}{2} + \mu(l)\mu(l+q) \right)}{(l+q)^2 - l^2} \frac{M(f(l+q) - f(l))^2 (2l^- + q^2\alpha)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}
\end{aligned} \tag{3.23}$$

This function is symmetric under the $\alpha \rightarrow 1 - \alpha$ transformation. In Figure 7 the distribution amplitude is shown for several values of Q^2 .

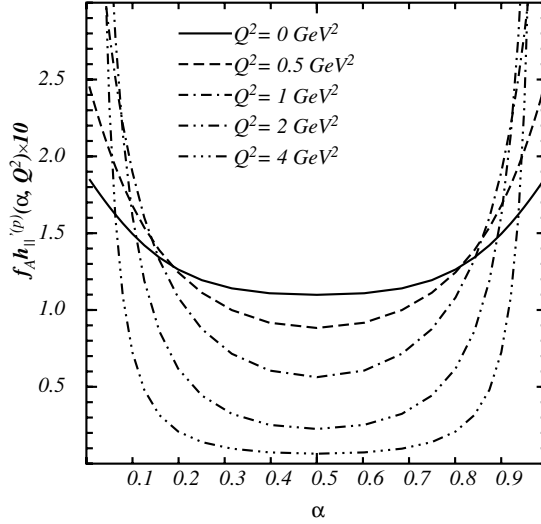


Figure 7. Distribution amplitude $h_{||}^{(p)}(\alpha)$ for several values of Q^2 .

3.7 Evaluation of $g_3(\alpha, q^2)$

In order to extract this function, we have to take a convolution of (3.12) with $p_\beta p_\mu$, which yields

$$\begin{aligned}
g_3(\alpha, q^2) &= \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} \right) \gamma_- \gamma_5 \psi \left(\frac{z}{2} n \right) J_-^5(x) \right| 0 \right\rangle \\
&= \frac{8N_c}{f_A} \int \frac{dl^- d^2 l_\perp}{(2\pi)^4} \left[\frac{2(l^-)^2 + l^- q^2}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
&\quad \left. + \frac{\left(\mu(l) \left(l^- + \frac{q^2}{2} \right) - \mu(l+q) l^- \right)}{((l+q)^2 - l^2)} \frac{M(f(l+q) - f(l))^2 \left(2l^- + \frac{q^2}{2} \right)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}
\end{aligned} \tag{3.24}$$

This function is symmetric under the $\alpha \rightarrow 1-\alpha$ transformation. In Figure 8 the distribution amplitude is shown for several values of Q^2 .

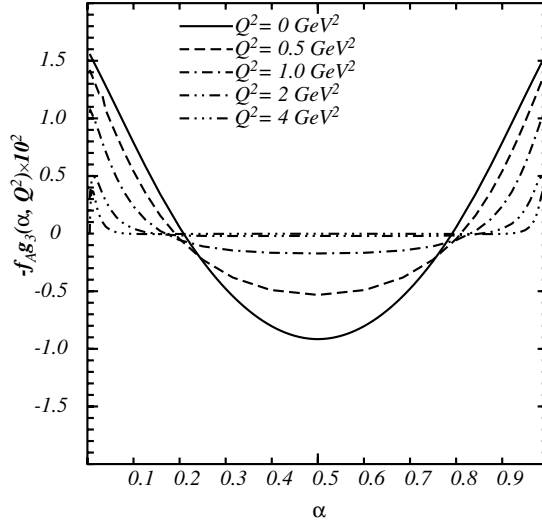


Figure 8. Distribution amplitude $g_3(\alpha)$ for several values of Q^2 .

3.8 Evaluation of $h_3(\alpha, q^2)$

In this section we evaluate the function

$$\begin{aligned}
h_3(\alpha, q^2) &= \frac{1}{4f_A} (g_{\beta\mu} p_\nu - g_{\beta\nu} p_\mu) \int d^4x e^{-iqx} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2} n \right) \sigma_{\mu\nu} \gamma_5 \psi \left(\frac{z}{2} n \right) J_\beta^5(x) \right| 0 \right\rangle \\
&= \frac{8N_c}{f_A} \int \frac{dl^- d^2 l_\perp}{(2\pi)^3} \left[\frac{l_- \mu(l+q) + (l_- + q_-) \mu(l)}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right. \\
&\quad \left. + \frac{1}{2} \frac{l_{\mu\perp}^2 q^2}{(l+q)^2 - l^2} \frac{M(f(l+q) - f(l))^2}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}
\end{aligned} \tag{3.25}$$

This function is antisymmetric under the $\alpha \rightarrow 1-\alpha$ transformation. In Figure 9 the distribution amplitude is shown for several values of Q^2 .

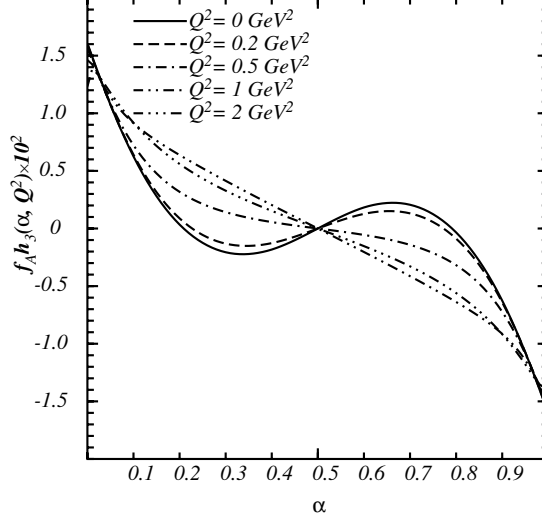


Figure 9. Distribution amplitude $h_3(\alpha)$ for several values of Q^2 .

4 Tests of PCAC

The PCAC hypothesis was proposed in the pre-QCD era [1–4] and has been intensively used as a phenomenological tool for describing the longitudinal part of the axial hadronic current at small virtualities. In operator form, the PCAC relation is

$$\partial_\mu J_\mu^{5,a} = f_\pi m_\pi^2 \phi^a, \quad (4.1)$$

where $J_\mu^{5,a}$ is the axial current; ϕ^a is the effective pion field; and f_π is the pion decay constant.

The pion DAs are defined as [25, 28],

$$\begin{aligned} \langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | \pi(q) \rangle &= i f_\pi \sqrt{2} \int_0^1 d\alpha e^{i(\alpha p \cdot y + \bar{\alpha} p \cdot x)} \times \\ &\times \left(p_\mu \phi_{2;\pi}(\alpha) + \frac{1}{2} \frac{z_\mu}{(p \cdot z)} \psi_{4;\pi}(\alpha) \right), \end{aligned} \quad (4.2)$$

$$\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | \pi(q) \rangle = -i f_\pi \sqrt{2} \frac{m_\pi^2}{m_u + m_d} \int_0^1 d\alpha e^{i(\alpha p \cdot y + \bar{\alpha} p \cdot x)} \phi_{3;\pi}^{(p)}(\alpha), \quad (4.3)$$

$$\begin{aligned} \langle 0 | \bar{\psi}(y) \sigma_{\mu\nu} \gamma_5 \psi(x) | \pi(q) \rangle &= -\frac{i}{3} f_\pi \sqrt{2} \frac{m_\pi^2}{m_u + m_d} \int_0^1 d\alpha e^{i(\alpha p \cdot y + \bar{\alpha} p \cdot x)} \times \\ &\times \frac{1}{p \cdot z} (p_\mu z_\nu - p_\nu z_\mu) \phi_{3;\pi}^{(\sigma)}(\alpha). \end{aligned} \quad (4.4)$$

These expressions can be bridged via Eq. (4.1) with the corresponding DAs of the axial current, Eqs. (3.17), (3.22), (3.23) and (3.24), resulting in,

$$f_A \Phi_{||}(\alpha, q^2 = m_\pi^2) = f_\pi^2 \sqrt{2} \phi_{2;\pi}(\alpha) \quad (4.5)$$

$$f_A g_3(\alpha, q^2 = m_\pi^2) = \frac{f_\pi^2 \sqrt{2}}{2} \psi_{4;\pi}(\alpha) \quad (4.6)$$

$$f_A h_{||}^{(t)}(\alpha, q^2 = m_\pi^2) = -\frac{\sqrt{2}}{3} \frac{f_\pi^2 m_\pi^2}{m_u + m_d} \phi_{3;\pi}^{(\sigma)}(\alpha) \quad (4.7)$$

$$f_A h_{||}^{(p)}(\alpha, q^2 = m_\pi^2) = \frac{2\sqrt{2} f_\pi^2 m_\pi^2}{m_u + m_d} \phi_{3;\pi}^{(p)}(\alpha) \quad (4.8)$$

The DAs of the pion Eqs. (4.2)-(4.4) were discussed in detail in [10, 11, 13]. The results of the calculations in the framework of the IVM the leading-order are presented in Appendix A.

Since both $\Phi_{||}(\alpha, q^2 = m_\pi^2) \approx \Phi_{||}(\alpha, 0)$ and $\phi_{2;\pi}(\alpha)$ are normalized to unity, we immediately conclude that

$$f_A = \sqrt{2} f_\pi^2 \quad (4.9)$$

Indeed, straightforward calculation shows that the relation (4.9) is satisfied with accuracy of about $\sim 5\%$, which is mainly related to the precision of numerical evaluation.

In Figure 10 we present the ratio of the left and right hand sides of Eqs. (4.5) - (4.8) in order to demonstrate how accurate is PCAC at different values of α . As we can see, PCAC is valid for all functions. The ratio $\phi_{2;\pi}/\Phi_{||}$ near the endpoints $\alpha \rightarrow 0, 1$, vanishes because both functions $\phi_{2;\pi}$ and $\Phi_{||}$ are suppressed at the endpoints, and as a result we have large numerical errors in those regions. For the ratio $g_3/\psi_{4;\pi}$ both DAs have very similar shapes. However, the position does not exactly match: numerical evaluation yields $\alpha_{\psi_{4;\pi}} \approx 0.786$, $\alpha_{g_3} \approx 0.791$. For all the other functions the PCAC relation is exact.

5 Conclusion

Partial conservation of axial current is a fundamental hypothesis, which involves the non-trivial dynamics of spontaneous breaking of chiral symmetry. This hypothesis has been tested in various process at low energy [3], and in the process of diffractive neutrino interaction at high energies [24, 25, 29]. We perform a direct test of the PCAC relation between the derivative of the axial current and the pion field, by comparing the distribution amplitudes, calculated within the light-cone dipole approach. Because PCAC essentially involves soft interactions, a perturbative QCD technique, usually used for calculation of the distribution amplitudes, cannot be employed. We derived several relations between the axial current and pion distribution amplitudes, and performed calculations in the framework of the instanton vacuum model. Our results confirmed that the based on PCAC relations are correct with high accuracy.

Acknowledgments

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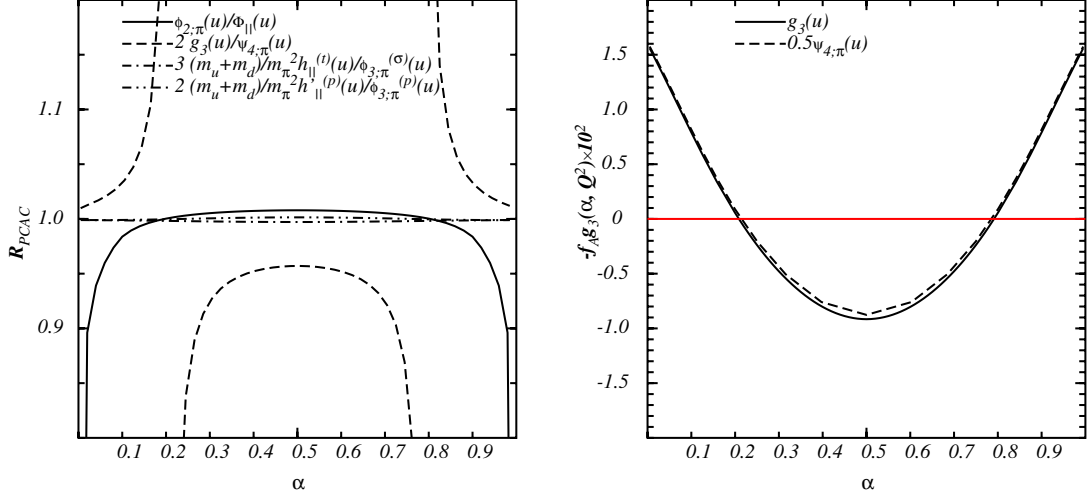


Figure 10. Left: Numerical verification of the PCAC relations within the IVM. According to PCAC, all the ratios should be equal to one. Right: comparison of g_3 and $\psi_{4;\pi}$. Because g_3 and $\psi_{4;\pi}$ have zeros at the points $\alpha_{g_3} \approx 0.5 \pm 0.291$ and $\alpha_{\psi_{4;\pi}} \approx 0.5 \pm 0.286$, the ratio plotted by dashed line in the left figure is significantly different from one near these points.

A Pion distribution amplitudes

The distribution amplitudes of the pion (4.2)-(4.4) and their evaluation in the framework of the IVM was performed and discussed in detail in [10, 11, 13, 15–17]. For the sake of completeness we present here the leading-order expressions for the pion distribution amplitudes.

$$\begin{aligned}
\phi_{2;\pi}(\alpha) &= \frac{1}{if_\pi\sqrt{2}} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \\
&\times \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n - \frac{\vec{r}_\perp}{2} \right) \hat{n} \gamma_5 \psi \left(\frac{z}{2}n + \frac{\vec{r}_\perp}{2} \right) \right| \pi(q) \right\rangle = \\
&= \frac{8N_c}{f_\pi\sqrt{2}} \int \frac{dl^- d^2l_\perp}{(2\pi)^4} e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} \\
&\times \left[Mf(l)f(l+q) \frac{\mu(l)\bar{\alpha} + \mu(l+q)\alpha}{(l^2 + \mu^2(l))((l+q)^2 + \mu^2(l+q))} \right]_{l^+ = -\alpha q^+}. \quad (A.1)
\end{aligned}$$

$$\begin{aligned}
\psi_{4;\pi}(\alpha) &= \frac{\sqrt{2}}{if_\pi} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \\
&\times \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n - \frac{\vec{r}_\perp}{2} \right) \hat{p} \gamma_5 \psi \left(\frac{z}{2}n + \frac{\vec{r}_\perp}{2} \right) \right| \pi(q) \right\rangle \\
&= \frac{16N_c}{f_\pi \sqrt{2}} \int \frac{dl^- d^2 l_\perp}{(2\pi)^4} e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} \\
&\times \left[Mf(l)f(l+q) \frac{\mu(l)(l_- + q_-) - \mu(l+q)l_-}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}. \tag{A.2}
\end{aligned}$$

$$\begin{aligned}
\phi_{3;\pi}^{(p)}(\alpha) &= \frac{1}{f_\pi \sqrt{2}} \frac{m_u + m_d}{m_\pi^2} \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \\
&\times \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n - \frac{\vec{r}_\perp}{2} \right) \gamma_5 \psi \left(\frac{z}{2}n + \frac{\vec{r}_\perp}{2} \right) \right| \pi(q) \right\rangle \\
&= \frac{8N_c}{f_\pi \sqrt{2}} \frac{m_u + m_d}{m_\pi^2} \int \frac{dl^- d^2 l_\perp}{(2\pi)^4} e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} Mf(l)f(l+q) \\
&\times \left[\frac{\mu(l)\mu(l+q) + l^2 + l \cdot q}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}. \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
\phi_{3;\pi}^{(\sigma)}(\alpha) &= \frac{3i}{2\sqrt{2}f_\pi} \frac{m_u + m_d}{m_\pi^2} (p_\mu n_\nu - p_\nu n_\mu) \int \frac{dz}{2\pi} e^{i(\alpha-0.5)p \cdot z} \\
&\times \left\langle 0 \left| \bar{\psi} \left(-\frac{z}{2}n - \frac{\vec{r}_\perp}{2} \right) \sigma_{\mu\nu} \gamma_5 \psi \left(\frac{z}{2}n + \frac{\vec{r}_\perp}{2} \right) \right| \pi(q) \right\rangle \\
&= -\frac{24N_c}{f_\pi \sqrt{2}} \frac{m_u + m_d}{m_\pi^2} \int \frac{dl^- d^2 l_\perp}{(2\pi)^4} e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} Mf(l)f(l+q) \\
&\times \left[\frac{q_+ l_- + \frac{q^2}{2} \alpha}{(l^2 + \mu^2(l)) \left((l+q)^2 + \mu^2(l+q) \right)} \right]_{l^+ = -\alpha q^+}. \tag{A.4}
\end{aligned}$$

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